

# Analyzing spatial coherence using a single mobile field sensor

P. A. Fridman

ASTRON, Oude Hoogeveensedijk 4, 7991PD Dwingeloo, The Netherlands

## Abstract

According to the Van Cittert-Zernike theorem the intensity distribution of a spatially incoherent source and the mutual coherence function of the light impinging on two wave sensors are related. It is the comparable relationship using a single mobile sensor moving at a certain velocity relative to the source which is calculated in this article. The autocorrelation function of the electric field at the sensor contains information about the intensity distribution. This expression could be employed in aperture synthesis.

## 1 Introduction

The mutual intensity and degree of coherence for light from an extended incoherent quasi-monochromatic source is explained by the Van Cittert-Zernike theorem [1]. This theorem is widely applied in optics and radio domains and in aperture synthesis, in particular[2]. Fig 1a shows the setup used in [1] to illustrate the Van Cittert-Zernike theorem. A screen  $\mathcal{A}$  is illuminated by an extended quasi-monochromatic spatially incoherent source  $\sigma$ . The source  $\sigma$  occupies a fragment of a plane parallel to screen  $\mathcal{A}$ . The linear dimensions of the source are small compared to the distance  $R$  between the source and the screen. The medium between the source and the screen is homogeneous and  $v$  is the velocity of light in the medium. Two points  $P_1$  and  $P_2$  are chosen on screen  $\mathcal{A}$  and the mutual intensity of light emitted by a source point  $S$  is calculated for these points. The angles between  $OO'$  and the lines connecting point  $S$  and  $P_1$  and  $P_2$  are small. The wave field created by  $S$  is represented by a complex analytical signal  $s(t) = E(t) \exp\{i[\Phi(t) - 2\pi\bar{\nu}t]\}$ , the envelope  $E(t)$  and the phase  $\Phi(t)$  vary slowly in comparison with  $\cos(2\pi\bar{\nu}t)$  and  $\sin(2\pi\bar{\nu}t)$ ,  $\bar{\nu}$  is the mean frequency. The complex envelope  $A(t) = E(t) \exp[i\Phi(t)]$  will be employed later. The frequency interval of width  $\Delta\nu$  is small compared to the mean frequency  $\bar{\nu}$ .

Under all these assumptions the mutual intensity, or the spatial coherence function of the field  $s(t)$  is [1] :

$$J(P_1, P_2) = \int_{\sigma} I(S) \frac{\exp\{i\bar{k}[R_1(S) - R_2(S)]\}}{R_1(S)R_2(S)} dS, \quad (1)$$

where  $R_1(S)$  and  $R_2(S)$  denote the distance between a typical source point  $S$  and the points  $P_1$  and  $P_2$  and  $\bar{k} = 2\pi\bar{\nu}/v = 2\pi/\bar{\lambda}$  is the wave number of the medium,  $\bar{\lambda}$  is the mean wavelength and  $I(S)$  is the intensity per unit area of the source. Eq. (1) is invertible, within reasonable limits, and is the basic equation in aperture synthesis [2]. In the simplest case the first wave sensor (receiver) is positioned at the fixed point  $P_1$ , while the second sensor is positioned in series at  $P_2$  and other points so as to obtain several samples of the spatial coherence function. Details of the proof of Eq. (1) are not included at this point but they will be reproduced in the calculation of mutual intensity for the setup in Fig. 1b which is the objective of this article.

## 2 Spatial coherence measured on a moving receiver

The difference between Fig. 1a and b is the following. A receiver in Fig. 1b is initially placed at point  $P_1$  and moves in the direction of point  $P_2$  with velocity  $V$ . The signal  $s(t)$  is processed in *real time*, or recorded and later processed *off-line* with the aim of calculating the autocorrelation function

$$J(P, \tau) = \langle s(t)s^*(t + \tau) \rangle, \quad (2)$$

where the sharp brackets denote the time average and the raised asterisk indicates the complex conjugate. Now  $J(P, \tau)$  can be calculated using the approach applied in [1] for Eq. (1). The source is divided into  $M$  elements  $d\sigma_1, d\sigma_2 \dots d\sigma_M$  centered on points  $S_1, S_2, \dots S_M$ , the linear dimensions of the elements are smaller than the mean wavelength  $\bar{\lambda}$ . Let  $s_m(t)$  and  $s_m(t + \tau)$  be the complex signals at point  $P_1$  at moment  $t$  and at the receiver at moment  $t + \tau$  when it reaches point  $P_2$  moving with velocity  $V$ , respectively. The total signals at these moments are

$$s(t) = \sum_m s_m(t), s(t) = \sum_m s_m(t + \tau), \quad (3)$$

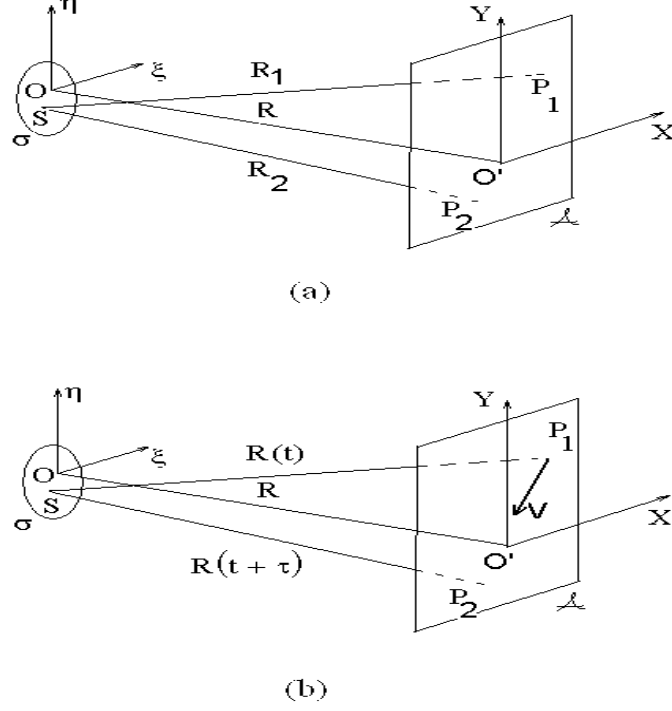


Figure 1: Setup for the Van Cittert-Zernike theorem for: a) two sensors; b) one moving sensor.

and the correlation function is

$$J(P, \tau) = \langle s(t) s^*(t + \tau) \rangle = \sum_m \langle s_m(t) s_m^*(t + \tau) \rangle + \sum_{m \neq n} \sum \langle s_m(t) s_n^*(t + \tau) \rangle \quad (4)$$

The second sum is equal to zero due to the incoherence of the signals from different source elements  $s_m$  and  $s_n, m \neq n$ . Let  $R_{m,P_1}$  and  $R_{m,P_2}(\tau)$  be the distances of the source element  $d\sigma_m$  from points  $P_1$  and  $P_2$ , respectively. So the partial signals are

$$s_m(t) = A_m \left[ t - \frac{R_{m,P_1}}{v} \right] \frac{\exp[-i2\pi\bar{\nu}(t - \frac{R_{m,P_1}}{v})]}{R_{m,P_1}},$$

$$s_m(t + \tau) = A_m \left[ t + \tau - \frac{R_{m,P_2}(\tau)}{v} \right] \frac{\exp\{-i2\pi\bar{\nu}[t + \tau - \frac{R_{m,P_2}(\tau)}{v}]\}}{R_{m,P_2}(\tau)} \quad (5)$$

The averaged product  $\langle s_m(t)s_m^*(t+\tau) \rangle$  is

$$\langle s_m(t)s_m^*(t+\tau) \rangle = \left\langle A_m(t)A_m^*(t+\tau - \frac{\Delta R_m(\tau)}{v}) \right\rangle \frac{\exp\{i2\pi\bar{\nu}[\tau - \frac{\Delta R_m(\tau)}{v}]\}}{R_{m,P_1}R_{m,P_2}(\tau)} \quad (6)$$

where  $\Delta R_m(\tau) = R_{m,P_1} - R_{m,P_2}(\tau)$ . The first factor in Eq. (6) is a complex correlation function of the envelope  $A_m(t)$ , which will be denoted as  $C_m[\tau - \frac{\Delta R_m(\tau)}{v}]$ . Quantity  $\langle s_m(t)s_m^*(t+\tau) \rangle$  is the correlation function corresponding to the source element  $d\sigma_m$ . Integration over the whole source  $\sigma$ , i.e., the transition to continuum in Eq. (4) gives

$$J(P, \tau) = \int_{\sigma} I(S)C[\tau - \frac{\Delta R(\tau, S)}{v}] \frac{\exp\{i2\pi\bar{\nu}[\tau - \frac{\Delta R(\tau, S)}{v}]\}}{R_{P_1}(S)R_{P_2}(S, \tau)} dS. \quad (7)$$

Let  $(\xi, \eta)$  be the coordinates of a source point  $S$ , referred to axes at  $O$ , and let  $(X_0, Y_0)$  and  $(X_\tau, Y_\tau)$  be the coordinates of  $P_1$  and  $P_2(\tau)$ , referred to the parallel axes at  $O'$ , Fig.1b. The distance  $OO'$  is equal to  $R$ . Then

$$R_{P_1}(S) = \sqrt{(X_0 - \xi)^2 + (Y_0 - \eta)^2 + R^2} \simeq R + \frac{(X_0 - \xi)^2 + (Y_0 - \eta)^2}{2R}. \quad (8)$$

The same approximation is valid for  $R_{P_2}(S, \tau) = R + \frac{(X_\tau - \xi)^2 + (Y_\tau - \eta)^2}{2R}$  and the difference  $\Delta R(\tau, S)$  is

$$\Delta R(\tau, \xi, \eta) \simeq \frac{(X_0^2 + Y_0^2) - (X_\tau^2 + Y_\tau^2)}{2R} - \frac{(X_0 - X_\tau)\xi + (Y_0 - Y_\tau)\eta}{R}. \quad (9)$$

$R_{P_1}(S)$  and  $R_{P_2}(S, \tau)$  in the denominator of the integral in Eq. (7) may be approximated by  $R$ . Using the notations

$$\frac{X_0 - X_\tau}{R} = p_\tau, \frac{Y_0 - Y_\tau}{R} = q_\tau, \psi(\tau) = 2\pi\bar{\nu}\tau + \frac{\bar{k}[(X_0^2 + Y_0^2) - (X_\tau^2 + Y_\tau^2)]}{2R}, \quad (10)$$

we get

$$J(P, \tau) = \frac{e^{i\psi(\tau)}}{R^2} \int_{\sigma} I(\xi, \eta)C[\tau - \frac{\Delta R(\tau, \xi, \eta)}{v}] \exp[-i\bar{k}(p_\tau\xi + q_\tau\eta)] d\xi d\eta. \quad (11)$$

Analyzing Eq. (11) we see that it represents a Fourier transform of the intensity function of the source (as in the Van Cittert-Zernike theorem),

but there is a weighting factor  $C(\tau)$  in the integral which must be taken into account when an inverse Fourier transform is performed to find  $I(\xi, \eta)$ . Another substantial feature is that with a moving receiver we get a continuum of baselines between points  $P_1$  and  $P_2$ , i.e., there is a scan of baselines in the interval  $[0, 0; p_\tau, q_\tau]$ . In theory, it is possible to capitalize on this property during aperture synthesis procedure [3],[4].

## References

- [1] M. Born and E. Wolf , *Principles of Optics* (Pergamon Press, Oxford, 6th ed., 1980), pp. 508-510.
- [2] *Synthesis Image in Radio Astronomy*, G. B. Taylor, C. L. Carilli, and R. A. Perley, eds. (ASP Conference Series, Vol. 180, 1999).
- [3] P. A. Fridman, , *Perspective on Radio Astronomy: Technologies for Large Antenna Arrays*, ed. A.B. Smolders and M.P. Haarlem, ASTRON, Dwingeloo, The Netherlands, pp. 277-283 (2000), <https://www.astron.nl/documents/conf/technology/tech37w.pdf>
- [4] P. A. Fridman, *Trans. of the IEEE, Antennas and Propagation*, v.51, n.7, 1658-1662, July 2003.